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UNSTEADY PROCESSES IN SOLID PROPELLANT COMBUSTION

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The purpose of this work is to develop a theoretical analysis of unsteady processes in solid propellant combustion, particularly combustion stability and extinction by rapid depressurization. It is assumed that the solid decomposes at its surface by a pyrolysis law, and the gaseous decomposition products react exothermically following an Arrenius law. The nondimensional activation energy of the gas phase reaction is considered to be large so that the reaction takes place in a thin zone where the temperature is close to the flame temperature. The gas phase is assumed to be quasisteady and under this condition the equations

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in the gas phase are solved in the limit of high activation energies. The analysis of the energy equation in the condensed phase shows that the characteristic response time of the solid to gas phase perturbations is large compared to the characteristic residence time in the heat-up zone of the solid, their ratio being of the order the nondimensional activation energy in the gas phase reaction. A stability criterion has been obtained, resulting that for large values of the nondimensional activation energy of the pyrolysis law there is always unstable behaviour. For the stable case the dynamic extinction problem is being presently investigated.

ABSTRACT

The purpose of this work is to develop a theoretical analysis of unsteady processes in solid propellant combustion, particularly combustion stability and extinction by rapid depressurization. The interest in understanding combustion stability stems from the fact that it may lead to inefficient operation in rocket motors. Extinction by depressurization is of interest in order to design rocket engines for space application with stop-restart capability.

It is assumed that the solid decomposes at its surface by a pyrolysis law, and the gaseous decomposition products react exothermically following an Arrhenius law. The nondimensional activation energy of the gas phase reaction is considered to be large so that the reaction takes place in a thin zone where the temperature is close to the flame temperature.

The gas phase is assumed to be quasisteady and under this condition the equations in the gas phase are solved in the limit of high activation energies yielding conditions for the burning rate and heat flux to the solid. The analysis of the energy equation in the condensed phase shows that the characteristic response time of the solid to gas phase perturbations is large compared to the characteristic residence time in the heat-up zone of the solid, their ratio being of the order of the nondimensional activation energy of the pyrolysis law there is always unstable behaviour. For the stable case the dynamic extinction problem is being presently investigated. Preliminary results seem to give an extinction criteria for step-like pressure changes.

Future work will include heat losses and will calculate higher order terms in the high activation energy limit. Possible retention of unsteady effects in the gas phase is also contemplated.

N O M E N C L A T U R E

B	Preexponential factor
c	Specific heat
D	Diffusion coeficient
E	Activation energy
E'	Dimensionless activation energy
l	Dimensionless heat flux at the surface, see Eq.(14)
L	Heat of vaporization
L_e	Lewis number
m	Burning rate
n	Exponent of fuel mass fraction in chemical reaction rate
p	Pressure
Q	Heat released per unit mass of fuel
R	Universal gas constant
T	Temperature
t	Time

t_c Characteristic time, see Eq.(43)

u Velocity

x Space coordinate

Y Fuel mass fraction

y Dimensionless space coordinate

α Thermal difussivity

γ Dimensionless steady state surface temperature, see Eq.(38)

ε Dimensionless inverse activation energy

θ Nondimensional temperature

λ Thermal conductivity

μ Dimensionless burning rate

ξ Nondimensional space coordinate

π Nondimensional pressure history

ρ Density

τ_1 Dimensionless time, see Eq.(19)

τ Dimensionless time, see Eq.(30)

ω See Eq.(38)

SUBSCRIPTS

f Flame

g Gas

s Surface

0 Steady state conditions

∞ Infinity in the solid

1. Introduction

An understanding of the unsteady burning of solid propellants provides insight into such important problems as the pressurization, depressurization and stability of a solid-rocket motor.

Research in the stability of combustion in a rocket motor, has been active over the past twenty years, since irregular pulses in chamber pressure were observed to develop[6] instead of the expected smooth pressure-time history. These irregular pulses are generally accompanied by more regular, small-amplitude, pressure oscillations, with frecuencies of the order of the natural vibrational frecuencies of sound waves in the chamber. Combustion instability leads to inefficient operation of rocket motors and even to mechanical failure of the propellant.

A considerable theoretical effort has been devoted towards the understanding of this phenomenon. However, there are so many different effects that may influence the stability of a burning solid as to prevent a consensus on the theoretical description of the phenomenon.

An excellent review of the analyses of the small amplitude pressure oscillations has been developed by Culick[7]. These analyses calculate the admittance function of the burning surface which gives the burning rate response to a pressure disturbance, indicating therefore wether pressure oscillations are amplified or attenuated. Most of the analyses assume the gas phase to be quasisteady, in the sense that the gas phase adjusts very quickly to changes in conditions when compared to the response of the solid, and differ mainly in the assumptions used to calculate the heat transfer from the gas to the solid phase. However as Culick points out, the majority of the results lead to the same two-parameter form of the admittance function, with different definitions of the two parameters. A stability boundary in the space with coordinates the two parameters

entering the admittance function, is defined as the curve where the admittance function becomes infinite, so that a small pressure change causes a large fluctuation in burning rate. Points above this stability boundary produce unstable solutions and points below this boundary provide stable steady solutions.

Interest in the response of a burning solid to an externally applied pressure variation stems from the possibility of extinction by a rapid pressure decay[8]. This dynamic extinction process is useful in order to design solid propellant rockets with stop-restart capabilities, of interest in connection with space applications.

Nearly all the theoretical models developed to explain dynamic extinction [9-13] invoke some kind of quasisteady approximation for the gas-phase. However, as pointed out in reference (12) some of these analyses[9-11] have interpreted incorrectly this assumption by implying that the heat feebback from the flame to the solid is a steady state function of the instantaneous pressure only.

The present paper is a first step towards analyzing unsteady processes in solid propellant burning by means of asymptotic techniques based on the assumption that the nondimensional activation energy of the gas-phase reaction is large. Attention is paid both to the problem of the stability of the steady state and to the response of the burning propellant to an externally imposed pressure variation.

We consider a one-dimensional model in which a condensed material gasifies by a rate-controlled surface process and then reacts in the gas phase. This gas phase reaction is described by an Arrhenius law, and we consider the limit in which the nondimensional activation energy is large. The quasisteady assumption is used to describe the gas-phase, so that we may use the results obtained by Williams[3] and Buckmaster et al[4], when analyzing the quasisteady burning of a solid in the limit of high activation energy. These analyses yield the

burning rate and the heat feedback to the solid as functions of the pressure and the flame temperature, and these relations are then used to analyze the unsteady response of the condensed phase. It is found that the characteristic response time of the solid is large of the order of the nondimensional activation energy, so that the temperature profiles are quasisteady is first approximation and the transient term is a perturbation of the quasisteady solution which may be calculated from the quasisteady profiles. In this way a differential equation is derived which describes the evolution with time of the burning rate as a function of the nondimensional surface temperature, the nondimensional pyrolisis activation energy and the pressure-time history. The analysis of this equation yield the stability condition of the steady state solution, and the response of the solid to an imposed pressure variation.

2. Formulation

We consider a one-dimensional model with the solid occupying the half space $x < 0$ and the gaseous phase the region $x > 0$. For convenience, the origin is fixed at the surface of the regressing solid. Figure 1 is a schematic representation of the process and shows the effects accounted for in this study.

Surface gasification is assumed to occur by an irreversible pyrolisis process which is described by an Arrhenius law. A one step over-all exothermic reaction takes place in the (premixed) gas. The present model has been used because it has been successfull in describing the steady-state deflagration of several propellants.

The equation of conservation of momentum reduces to the statement that the pressure is approximately uniform throughout the region treated but varies with time[1], and we will also use the well justified assumptions that the work associated with viscous and external forces is negligible and we will use Fick's law to calculate the diffusion velocities.

With these assumptions, the conservation equations of mass, concentration and energy in the condensed phase and in the gas phase, become respectively.

$$\frac{\partial \rho_g}{\partial t} + \frac{\partial(\rho_g u)}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial Y}{\partial t} + u\frac{\partial Y}{\partial x} - \frac{1}{\rho_g}\frac{\partial}{\partial x}\left(\rho_g D_g \frac{\partial Y}{\partial x}\right) = - B p^n Y^n \exp\left(-\frac{E_g}{RT}\right) \qquad (2)$$

$$c_g \frac{\partial T}{\partial t} + c_g u \frac{\partial T}{\partial x} - \frac{1}{\rho_g}\frac{\partial}{\partial x}\left(\lambda_g \frac{\partial T}{\partial x}\right) - \frac{1}{\rho_g}\frac{\partial p}{\partial t} = QB\, p^n Y^n \exp\left(\frac{E_g}{RT}\right) \qquad (3)$$

$$\rho c \frac{\partial T}{\partial t} + mc \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) \tag{4}$$

where all symbols are defined in the nomenclature.

These equations, together with the equation of state should determine ρ_g, u, Y, T, T_g.

The boundary conditions are

$$T(t, 0^+) = T(t, 0^-) = T_s \tag{5}$$

$$-\lambda \frac{\partial T}{\partial x}\Big|_{s^-} + \lambda_g \frac{\partial T}{\partial x}\Big|_{s^+} = m\left[(c_g - c)T_s + L\right] \tag{6}$$

$$\rho_g D_g \frac{\partial Y}{\partial x}\Big|_{s^+} = m(Y_{s^+} - 1) \tag{7}$$

$$T = T_\infty \quad \text{at} \quad x \to -\infty \;, \quad Y = 0 \;, \quad T = T_f \quad \text{at} \quad x \to \infty \tag{8}$$

The pyrolisis process is assumed to follow an Arrhenius law

$$m = B' \exp(-E/RT_s) \tag{9}$$

where m is the mass flux relative to the burning surface, and E the activation energy of the pyrolisis process.

3. Gas-phase analysis

In most studies of unsteady solid propellant burning, the gas-phase is assumed to be quasisteady in the sense that the response time in the gas is short compared to the response time of the condensed phase. The ratio of these characteristic times is of the order of the ratio of the thermal responsivities of solid and gas, which is usually small[2]. The quasisteady assumption will not be valid when analyzing the response of a burning solid to

very high frecuency pressure oscillations[1].

When the gas-phase is considered to be quasisteady, the analysis is greatly simplified since the conservation equations are uncoupled. All the time derivate terms may be neglected so that equation (1) reduces to $\rho_g u = m$. In addition the dp/dt term in equation (3) may be neglected. Under these conditions the gas-phase equations may be solved in the limit of high activation energy of the gas-phase reaction. This quasisteady solution has been derived by Williams[3] in the case of Lewis number unity and by Buckmaster et al[4]., for arbitrary Lewis number, Le.

Rather than repeat those analyses we will only state the results needed in the following sections. The reader is referred to reference 3 and 4 for details. The analysis in reference (4) considers Le constant but arbitrary. In the limit of high activation energy, it is found that the reaction term is only significant in a thin flame where T is close to T_f. In terms of the nondimensional variables

$$\theta = c_g T/Q_g \qquad \xi = \int_0^x (m/\alpha)dx \tag{10}$$

the solution outside of the flame sheet becomes simply

$$\left.\begin{aligned} \theta &= \theta_s - 1 + 1\exp(\xi) \\ Y &= 1 - \exp\{L_e(\xi - \xi_f)\} \end{aligned}\right\} \quad \text{for } \xi < \xi_f \tag{11}$$

$$\begin{aligned} \theta &= \theta_f \\ & \qquad \text{for} \quad \xi > \xi_f \\ Y &= 0 \end{aligned} \tag{12}$$

where ξ_f, the location of the flame sheet, is given to leading order by

$$\xi_f = \ln(1/1) \tag{13}$$

The parameter

$$l = \frac{d\theta}{d\xi}\bigg|_s \tag{14}$$

is the dimensionless heat conducted out of the gas at the interface. The effect of the solid and the pyrolisis are concealed in l. An overall energy balance in the gas phase, provides a relationship between θ_s, θ_f and l.

$$l = 1 + \theta_s - \theta_f \tag{15}$$

Since the flame sheet must lie in the gas, equation (13) provides limitations in the possible values of l

$$0 < l = 1 + \theta_s - \theta_f < 1 \tag{16}$$

When $l \to 0$ the flame sheet moves to infinity and no heat reaches the condensed phase from the flame. When $l \to 1$ the flame sheet approaches the surface and all the heat generated at the flame goes to the solid.

Solutions for $l \to 0$ and $l \to 1$ are presented in reference (4)

The reaction zone is located in the vicinity of ξ_f, where $(\xi - \xi_f)$ is of order θ_f^4/E'^2. The parameter E' is the nondimensional activation energy $E' = E_g c_g/RQ$. In the reaction zone the temperature differs from the flame temperature by a small quantity of order T_f^2R/E_g. To leading order, only the reactive and diffusive terms are important in this zone. Solution of the energy equation with the appropriate matching conditions to the frozen solution outside of the flame sheet, provides the burning rate eigen-value.

In this way an expression for the burning rate is obtained

involving the pressure and the flame temperature, namely

$$m=\left|2\Gamma(n+1)\alpha B\theta_f^{2n+2}/E'^{n+1}\right|^{1/2}\exp\left(-\frac{E'}{2\theta_f}\right)p^{n/2} \quad (17)$$

For large activation energy the effect of the exponential term is dominant so that the square root term may be taken as constant when analyzing small changes in the flame temperature. Equation (17) coincides with the Denison-Baum formula[5].

4. Condensed Phase Analysis

The characteristic time in the condensed phase is short compared with the characteristic time in the gas phase[2]. Therefore, the condensed phase should not be considered quasisteady. However, a brief description of the quasisteady solution will be presented before considering the unsteady analysis.

In terms of the nondimensional variables for the solid

$$\theta = \frac{T-T_\infty}{T_{so}-T_\infty} \qquad y = \frac{m_o c}{\lambda}x \quad (18)$$

$$\tau_1 = \frac{m_o^2 c}{\lambda\rho}t \qquad \mu = \frac{m}{m_o} \quad (19)$$

Eq. (4) may be written

$$\frac{\partial\theta}{\partial\tau_1} + \mu\frac{\partial\theta}{\partial y} = \frac{\partial^2\theta}{\partial y^2} \quad (20)$$

The subscribt o refers to the initial, steady condition. The boundary condition (6) may be written, ...

by using Eqs. (14) and (15) as

$$\left.\frac{\partial\theta}{\partial y}\right|_s = \frac{\mu}{c(T_{so}-T_\infty)}\left[cT_\infty - c_g T_f + Q - L + c\theta_s(T_{so}-T_\infty)\right] \tag{21}$$

To solve the quasisteady problem, we may neglect the time derivative term in Eq. (20). The solution to Eq. (20) is simply

$$\theta = \theta_s \exp(\mu y) \tag{22}$$

so that from Eq. (21).

$$c_g T_{fo} = cT_\infty + Q - L \tag{23}$$

This equation can also be obtained by an overall energy balance and shows that the flame temperature is constant in a quasisteady process. For a given T_∞, Eq. (23) gives the value of T_{fo} which can be used in the Denison-Baum formula, Eq. (17), to calculate m. The pyrolisis law, Eq. (9),yields T_s, so that Eqs. (11), (12) and (22) describe the complete temperature profile.

Figure (2) shows the burning rate m under steady conditions; as a function of pressure for a fixed value of T_∞ and therefore of T_{fo}. This curve is calculated by using Eqs. (17) and (23).

The surface temperature T_s is a parameter along the curves of figure (2), since it is related to the burning rate m_o through the pyrolisis law. However, T_s is limited by the inequalities (16) so that only a portion of the curve applies. At P_∞ the flame sheet has moved off to infinity. At P_s the flame sheet has reached the surface.

We will now analyze the evolution with time of an initially steady temperature profile during an unsteady process. After some manipulations, Eqs. (20) and (21) provide the following relationships

$$\left.\frac{\partial\theta}{\partial y}\right|_s - \mu\theta_s = \int_{-\infty}^{o} \frac{\partial\theta}{\partial\tau_1}\, dy = \mu\, \frac{c_g}{c}\, \frac{T_{fo}-T_f}{T_{so}-T_\infty} \qquad (24)$$

where use has been made of Eq. (23). The left hand side of the preceding equation represents the difference between the heat flux existing during an unsteady process and the one existing if the process was quasisteady. We will consider unsteady processes that result in changes of order unity in the burning rate with respect to the one existing under quasisteady conditions. Equation (17) shows that in the limit of high activation energy of the gas phase reaction, small changes in the flame temperature of order RT_f^2/E_g, produce variations of order unity in the burning rate. Therefore, the right hand side of Eq.(24) is small, so that the heat conducted to the solid during an unsteady process differs by a small quantity from the heat conducted if the process was quasisteady.

The second equality of Eq.(24) indicates that the variation with time of the heat content in the heat up zone of the solid is small of order RT_f^2/E_g. When this heat content decreases, the solid appears from the gas phase as a heat source, and therefore the flame temperature increases.

It is necessary to wait times of order E_g/RT_f^2 to produce changes of order unity in the heat content of the heat up zone in the solid. The characteristic response time of the solid is therefore long compared with the characteristic residence

time in the heat up zone.

The Denison-Baum formula, Eq. (17), yields

$$\mu = \left(\frac{T_f}{T_{fo}}\right)^{n+1} \left(\frac{P}{P_o}\right)^{n/2} \exp\left[\frac{E_g}{2RT_{fo}} - \frac{E_g}{2RT_f}\right] \tag{26}$$

which in the limit of high values of $E_g/2RT_{fo}^2$ becomes

$$\frac{\mu}{\pi} = \exp\left\{\frac{E_g(T_f - T_{fo})}{2RT_{fo}^2}\right\} \tag{27}$$

where

$$\pi = \left(\frac{P}{P_o}\right)^{n/2} \tag{28}$$

Let's define a small parameter ε, as

$$\varepsilon = \frac{c_g}{c} \frac{2RT_{fo}^2}{E_g(T_{so} - T_\infty)} \tag{29}$$

and introduce as nondimensional time variable

$$\tau = \varepsilon \tau_1 \tag{30}$$

Equation (27) may be used to express Eq. (20) and the boundary condition (24) in terms of the new time variable τ, as

$$\varepsilon \frac{\partial \theta}{\partial \tau} + \mu \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \tag{31}$$

$$\left.\frac{\partial \theta}{\partial y}\right|_s = \mu\left[\theta_s - \varepsilon \ln \frac{\mu}{\pi}\right] \tag{32}$$

Introducing the expansions

$$\theta = \theta_o + \varepsilon\, \theta_1 + \,.. \tag{33}$$

$$\mu = \mu_o + \varepsilon\, \mu_1 + \,.. \tag{34}$$

in Eqs. (31) and (32), expanding and collecting like powers of ε the following equations defining θ_o and θ_1 are obtained

$$\mu_o \frac{\partial \theta_o}{\partial y} = \frac{\partial^2 \theta_o}{\partial y^2} \quad , \quad \left.\frac{\partial \theta_o}{\partial y}\right|_s = \mu_o \theta_{os} \tag{35}$$

$$\left.\begin{aligned} &\frac{\partial \theta_o}{\partial \tau} + \mu_o \frac{\partial \theta_1}{\partial y} + \mu_1 \frac{\partial \theta_o}{\partial y} = \frac{\partial^2 \theta_1}{\partial y^2} \\ &\left.\frac{\partial \theta_1}{\partial y}\right|_s = \mu_1 \theta_{os} + \mu_o \theta_{1s} - \mu_o \ln \frac{\mu_o}{\pi} \end{aligned}\right\} \tag{36}$$

The solution to Eq. (35) is the quasisteady solution, Eq. (2
The value of the surface temperature may be written in terms of th burning rate μ, through the pyrolisis law, Eq. (9), resulting

$$\theta_{os} = \frac{\frac{\omega}{\gamma - 1} \ln\mu + 1}{1 - \omega \ln \mu} \tag{37}$$

where

$$\gamma = \frac{T_{so}}{T_\infty} \quad , \quad \omega = \frac{RT_{so}}{E} \tag{38}$$

Integrating Eq. (36) from $-\infty$ to the surface, the following expression is derived

$$\int_{-\infty}^{o} \frac{\partial \theta_o}{\partial \tau}\, dy = \cancel{-\mu_o \theta_{1s}} \cancel{-\mu_1 \theta_{os}} \cancel{+\mu_1 \theta_{os}} \cancel{+\mu_o \theta_{1s}} - \mu_o \ln \frac{\mu_o}{\pi} \tag{39}$$

The quasisteady solution, Eq. (22), together with Eq.(37) may be used to calculate the integral in the left hand side of the preceding equation, thus obtaining

$$\frac{d\mu_o}{d\tau} = - \frac{(\gamma-1)(1-\omega \ln\mu_o)^2 \mu_o^3}{\gamma\omega-\gamma+1+\omega(\gamma-2)\ln\mu_o+\omega^2\ln^2(\mu_o)} \ln(\frac{\mu_o}{\pi}) \quad (40)$$

which describes the evolution with time of the burning rate during an unsteady process, in terms of two parameters γ and ω. For a quasisteady evolution $d\mu_o/d\tau$ vanishes and therefore $\mu_o = \pi$.

The value of the burning rate, μ_o, obtained from Eq. (40), may be used in the pyrolisis law to deduce the surface temperature history, and through Eq. (22) the complete temperature profile.

5. Stability Analysis

In this section an analysis is presented of the stability of the steady deflagration of a solid which undergoes an Arrhenius type gas phase reaction with large activation energy.

The pressure is considered to be constant, so that $\pi=1$. The burning rate of the steady solution is m_o, so that initially $\mu_o=1$. Let's assume that at $\tau = 0$ a perturbation changes the value of the burning rate so that $\mu_o = 1+\mu'$. Eq. (40) shows that

$$\frac{d\mu'}{d\tau} = - \frac{\mu'}{\frac{\gamma\omega}{\gamma-1} - 1} \quad (41)$$

Therefore the stability of the steady state solution depends on the value of the parameter $\frac{\gamma\omega}{\gamma-1}$. If this parameter is greater than one, the solution is stable, being unstable when it is smaller than one. In the stable case the perturbations decrease

exponentially to zero with time, while in the unstable case the perturbations grow exponentially. The parameter

$$\frac{\gamma-1}{\gamma\omega} = \frac{E(T_{so}-T_{\infty})}{RT_{so}^{2}} = A \qquad (42)$$

is identical with the parameter A used by Denison and Baum[5] when analyzing the unstable burning of solid propellants. These authors develop a linearized analysis of the response of a burning solid to a pressure disturbance using a model which parallels the one used in the present analysis. The difference however, is that in our analysis we retain the nonlinear effects that were linearized in reference (5). In figure (1) of their paper, Denison-Baum show the stability boundary which separates regions of stable from regions of unstable burning. Two parameters A and α, define the stability of a given solution. The parameter α is essentially identical to our parameter ε, and in that figure (1) we observe that for $\alpha \to 0$ the solutions are stable for $A < 1$ and unstable for $A > 1$. Therefore our stability criterion coincides with the one of Denison and Baum.

The instability of the one dimensional deflagration may be clearly interpreted, when the activation energy of the pyrolisis is large (ω small). Let's assume that at a certain time a perturbation causes the burning rate to increase (decrease) with respect to its steady value. Since the activation energy of the pyrolisis is large, the surface temperature will remain nearly constant. However, the width of the heat up zone in the solid will decrease (increase), so that the total heat content of the solid will decrease (increase). The variation of the thermal energy of the solid produces an increase (decrease) in the flame temperature, as may be seen from Eq. (24), which will further increase (decrease) the burning rate. This selfaccelerating behaviour results in instability of the one-dimensional deflagration. When the activation energy

of the pyrolisis is not large, there are two effects which control the response of the burning solid to a perturbation in the burning rate. When the burning rate increases the width of the heat up zone decreases and the surface temperature increases. The decrease in width tends to decrease the heat content of the solid thus producing instability, and the increase in surface temperature tends to increase the heat content of the solid thus producing stability. The stability of the solution depends on which of these two effects dominates.

6. Response of a burning solid to pressure variations.

In this Section an analysis is presented of the response of a burning solid to an imposed pressure variation. The evolution with time of the burning rate is given by the solution of Eq. (40) with π being a known function of time defined by Eq. (28).
the nondimensional pressure

It was shown in section (4) that the characteristic response time of the solid is

$$t_c = \frac{\lambda\rho}{m_o^2 c_g} \frac{E_g(T_{so}-T_\infty)}{2RT_{fo}^2} \tag{43}$$

Therefore, if the characteristic time of pressure variation is small compared with it, the pressure variation function π is a step function. If the time of pressure variation is long compared to the characteristic time shown in Eq. (43) the response of the solid may be considered as a sequence of stationary states, and at any given time $\mu = \pi$.

The solution to Eq. (40) depends on the values of the parameters γ, ω and of the pressure time history.

To analyze the different types of solutions that may result from integration of Eq. (40), let's first study the singularities of that equation.

The numerator vanishes at

$$\mu = \exp(1/\omega) \quad , \quad \mu = \pi \quad , \quad \mu = 0 \tag{44}$$

and the denominator at

$$\ln \mu = - \frac{1}{2\omega} \{\gamma - 2 \mp \sqrt{\gamma(\gamma - 4\omega)}\} \tag{45}$$

In the present analysis we will only consider depressurization problems, so that π will decrease from 1 at time 0 to its final value π_F. In addition we will always assume that at the initial pressure the solution is stable, so that $\gamma\omega/(\gamma-1)$ is greater than one. The existence of singularities (where the denominator and numerator vanish simultaneously) depends on the values of γ, ω and π_f, and it is observed that in order to have real solutions to Eq. (45) it is necessary that

$$\gamma > 4\omega \tag{46}$$

In dimensional variables $E_s/RT_\infty > 1$ for the existence of roots.

It is observed that when the solution at the initial pressure is stable, both roots of Eq.(45) have the same sign. So that

$$\left.\begin{array}{ll} \gamma < 2 & \text{2 positive roots} \\ \gamma > 2 & \text{2 negative roots} \end{array}\right\} \tag{47}$$

and the denominator vanishes at two different values of μ, which

are either both greater than one or both smaller than one. In dimensional variables $T_{so} > 2T_{\infty}$ for the existence of two negative roots.

Since π is a decreasing function of time, which starts at 1 and goes to π_f, singularities will only exist when $\gamma > 4\omega$, $\gamma > 2$ and π_f smaller than the larger root of Eq. (45). When π_f is smaller than both roots there are two singularities, and when π_f lies between both roots there is only one singularity.

The condition for the steady state solution at the final pressure being stable, may be written

$$\frac{\gamma\omega}{(1-\omega\ln\pi_f)(\gamma-1+\omega\ln\pi_f)} > 1 \qquad (48)$$

The left hand side of this equation is represented in Figure (3) as a function of ln π_f. When ln π_f approaches the value $1/\omega$ ($T_s \to \infty$) the left hand side of Eq. (48) approaches ∞ , and the solution at the final pressure is stable. Analogously the solution is stable when ln π_f approaches the value $(1-\gamma)/\omega$ ($T_s \to 0$). Between these two values, the function has a minimum when $\ln\pi_f$ equals $(2-\gamma)/2\omega$ and depending on wether the value of the function at this minimum is larger or smaller than one, the solution will be stable for any final pressure or there will be a range of final pressures for which the solution will be unstable. The equation which gives the values of ln π_f where the function defined in Eq. (48) crosses to the unstable region coincides with Eq. (45).

Let's denote these two values (if they exist) by ln π_1 and ln π_2. We may conclude that if $\gamma < 4\omega$ the solution at any final pressure is stable. Also, if $\gamma > 4\omega$ and $\gamma < 2$, the solution is stable for any final pressure smaller than the initial pressure,

being unstable in a finite range of pressures larger than the initia pressure. Finally, if $\gamma > 4\omega$ and $\gamma > 2$ the solution is unstable in a range of values of final pressures smaller than the initial pressure. We may briefly summarize the different possible cases in a depressurization process during which the nondimensional pressure goes from the initial value of 1, to a final value $\pi_f < 1$.

a) $\gamma < 4\omega(\frac{4RT_\infty}{E} > 1)$: The steady state solution is stable at the initial, final and all intermediate pressures occurring during the process. Figure (4) shows the evolution with time of the burning rate for the case in which a step change in pressure takes place at time zero. Figure (5) shows the burning rate evolution for depressurization processes of the type

$$\pi = (1-a)e^{-b\tau} + a \qquad (49)$$

with different values of b.

b) $\gamma > 4\omega,\ \gamma < 2\ (\frac{4RT_\infty}{E} < 1,\ T_{so} < 2T_\infty)$.: This case is analogous to the preceding one. The steady state solution at the final pressure is more stable then the steady state solution at the initial pressure, so that a perturbation dies out more rapidly at the final than at the initial pressure.

c) $\gamma > 4\omega,\ \gamma > 2, \pi_f > \pi_1$: This case is analogous to case (a) with the solution at the initial pressure being more stable than the solution at the final pressure.

d) $\gamma > 4\omega, \gamma > 2, \pi_1 > \pi_f > \pi_2$: The solution is unstable at the final pressure, and Eq. (40) has a singularity at $\mu = \pi_1$, and τ_1 such that $\pi(\tau_1) = \pi_1$. The character of the singularity depends on the value of the slope of the pressure time function curve at π_1. When

$$\left|\frac{d\pi}{d\tau}(\tau=\tau_1)\right| < \frac{\gamma-1}{\omega}\;\frac{\mu_1^3(1-\omega\ln\mu_1)^2}{\sqrt{\gamma(\gamma-4\omega)}} \tag{50}$$

the singularity is a node, being a spiral when this inequality is not satisfied. Figure (6) shows the isoclines of Eq. (40) for the two possible cases, together with the solution with the initial condition $\mu = 1$. When the singularity is a node, the solution to Eq. (40) will be a smooth function approaching π_f for $\tau \to \infty$. However it is observed that for $\mu < \pi_1$ the solution enters a region of unstable states and any perturbation will grow exponentially with time. The one-dimensional flame is unstable, so that our solution is no longer valid. One may speculate that when the flame becomes unstable, some type of two-dimensional effect or other effects, will appear in order to stabilize the flame. When the singularity is a spiral, our solution ceases to be valid when μ approaches π_1, since the variation of burning rate with time becomes very rapid, so that it is no longer valid to assume that the time derivative term is only a perturbation of the quasisteady solution. A different analysis will be required to study the behaviour of the solution near $\mu=\pi_1$. However, since μ is decreasing very rapidly to zero, this condition may be identified as a dynamic extinction caused by a rapid depressurization process

e) $\underline{\gamma > 4\omega,\ \gamma > 2,\ \pi_f < \pi_2}$: The solution is stable at the initial and at the final pressure, but there is an intermediate range of pressures ($\pi_2 < \pi < \pi_1$) where the solution is unstable. There are two singularities of Eq. (40) at $\mu=\pi_1$, $\pi(\tau_1)=\pi_1$ and at $\mu=\pi_2$, $\pi(\tau_2)=\pi_2$. The singularity at τ_1 is a node when Eq.(50) is satisfied and a spiral in all other cases. The singularity at τ_2 is always a saddle. Figure (7) shows the isoclines for the two possible cases, and the solution of Eq. (40). The directions at the node, are given by

$$\frac{d\mu}{d\tau} = - \frac{1 \mp \sqrt{1+2M \left.\frac{d\pi}{d\tau}\right|_1}}{M} \tag{51}$$

where

$$M = \frac{2\omega\sqrt{\gamma(\gamma-4\omega)}}{(\gamma-1)\mu_1^3(1-\omega\ln\mu_1)^2} \tag{52}$$

with the minus sign in Eq. (51) corresponding to the principal direction. It is observed that the principal direction has a slope larger in absolute value than the slope of the pressure function π. The directions at the saddle are given by

$$\frac{d\mu}{d\tau} = \frac{1 \pm \sqrt{1-2M \left.\frac{d\pi}{d\tau}\right|_2}}{M} \tag{53}$$

where

$$M = \frac{2\omega\sqrt{\gamma(\gamma-4\omega)}}{(\gamma-1)\mu_2^3(1-\omega\ln\mu_2)^2} \tag{54}$$

so that one direction has positive slope and the other one negative slope. When the singularity at τ_1 is a node, the solution starting at $\mu = 1$ is the separatrix. This solution enters the unstable region through the node and exits by the saddle, reaching in this way the region of stable states. However any perturbation occurring when the value of μ lies in the unstable region will grow with time so that the one-dimensional flame will be unstable in this zone. Other effects not considered in this study may then appear to stabilize the flame. As in the previous case, if the singularity at τ_1 is a spiral, our analysis ceases to be valid when μ approaches π_1. The burning rate decreases very rapidly with time, suggesting the onset of dynamic extinction.

Figure (8) shows the stability regions as a function of the parameters γ and ω. At the initial point of a depressurization process ($\ln \mu = 0$), the steady state solution is stable or not depending on wether the value of ω is at the right or left of the intersection point of the γ curve with the ω axis respectively. The steady state solution at the final pressure of a depressurization process, has the same value of γ and ω but a different value of μ($\ln \mu < 0$). The stability of this final point depends on wether or not it is located at the right or left of the corresponding γ curve. It is observed that for $\gamma > 2$ there is a vertical tangent of the γ curves, and therefore it is possible to have depressurization processes such that the steady state solutions corresponding to the initial and final pressure are stable although there are intermediate pressures whose steady state solution is unstable.

7. Conclusions

We have developed an asymptotic analysis of the unsteady burning of a solid propellant in the limit of high activation energy The analysis shows that the ratio of the characteristic pressure and thermal response times of the solid, (see Eq. (43)) is of the order of the nondimensional activation energy of the gas phase reaction. By integrating the condensed phase energy equation, an equation is derived whose solution yields the evolution with time of the burning rate.

It is found that for $E(T_{so}-T_{\infty})/RT_{so}^2$ greater than one the steady state solution is dinamically unstable. However, it is not clear what happens when the one-dimensional, steady deflagration solution becomes unstable. T'ien[14], identifies the onset of instabilities with the occurrence of extinction, thus defining a low pressure dynamic deflagration limit, which does not coincide with the low pressure static deflagration limit. It is more likely however, that some nonlinear mechanism or tri-dimensional effect will prevent the growth of the oscillations and the occurrence of extinction, so that only the one-dimensional steady deflagration described in the present analysis will cease to exist, not the flame. The intrinsic instability of the one-dimensional deflagration is analogous to the problem of normal flame propagation which has been recently analyzed by Sivashinsky[15], showing that for large activation energy and Lewis number greater than one the solution becomes unstable.

The analysis has also been applied to study the response of a burning solid subjected to a depressurization process. The characteristic response time of the solid is given be Eq.(43), so that when the time of pressure variation is short compared with it, the depressurization process appears as a step change in pressure. We assume that the steady solution at the initial pressure is stable, and derive the conditions necessary for the steady solution to be stable at the final pressure and at all intermediate pressures. It

is shown that when the steady solution is stable at all pressures occuring during the process, the burning rate always approaches for longe times, the value corresponding to the steady solution at the final pressure. The burning rate does not adjust instantaneously to the applied pressure so that during a fast change in pressure the value of the burning rate is larger that the quasisteady value, approaching it for long time. The absence of dynamic extinction appears to coincide with the suggestion of T'ien[13], who considers that heat losses is the mechanism responsible for dynamic extinction.

From analysis of Eq.(40) which gives the time evolution of the burning rate, it is found that when the steady solution is stable at the initial pressure but becomes unstable at some lower pressure which is reached during the process, a singularity of Eq. (40) occurs when the pressure reaches the value at which the steady solution becomes unstable. The character of the singularity depends on the slope of the pressure-time curve at the singularity. When the slope is larger than a critical value defined in Eq.(50) the singularity is a spiral. The solution derived in the present analysis is valid until a certain time when the variation of the burning rate with time becomes infinite. From that time the transient term becomes of the order of the convective and diffusive terms and our analysis ceases to be valid. However, since the burning rate is rapidly decreasing towards zero, this condition may be identified with the onset of dynamic extinction. When the slope of the pressure-time curve is smaller than the critical value defined in Eq.(50), the singularity is a node. The burning rate solution in absence of disturbances will approach for long times its steady value corresponding to the final pressure. However, in the region of unstable states, any disturbance will be amplified so that the steady one-dimensional deflagration ceases to be a valid solution, and some other type of solution will exist.

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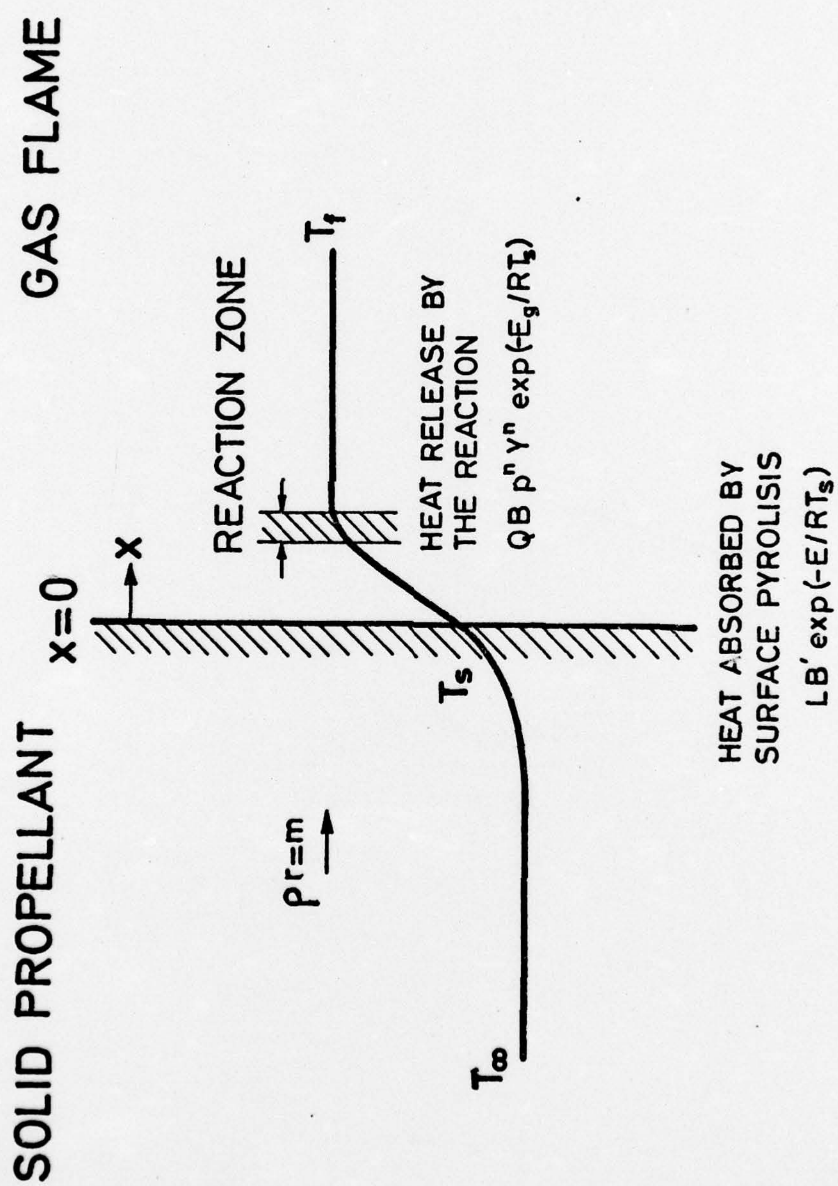
LIST OF FIGURES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GAS FLAME
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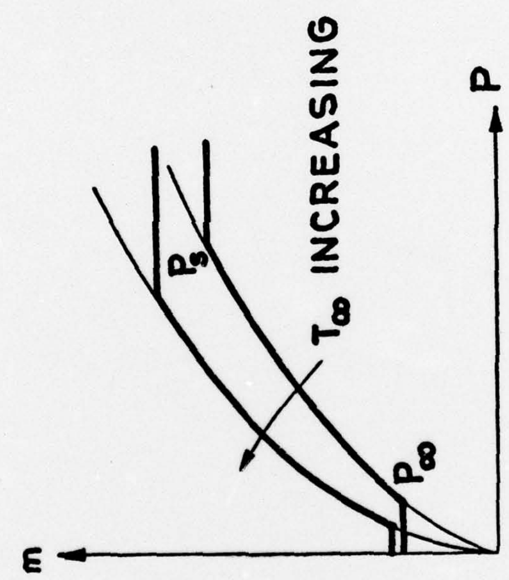
X
REACTION ZONE
T_f
T_s
T_∞
$\rho r = m$
HEAT RELEASE BY THE REACTION
$QB\, p^n Y^n \exp(-E_g/RT_f)$
HEAT ABSORBED BY SURFACE PYROLISIS
$LB' \exp(-E/RT_s)$

Figure 1



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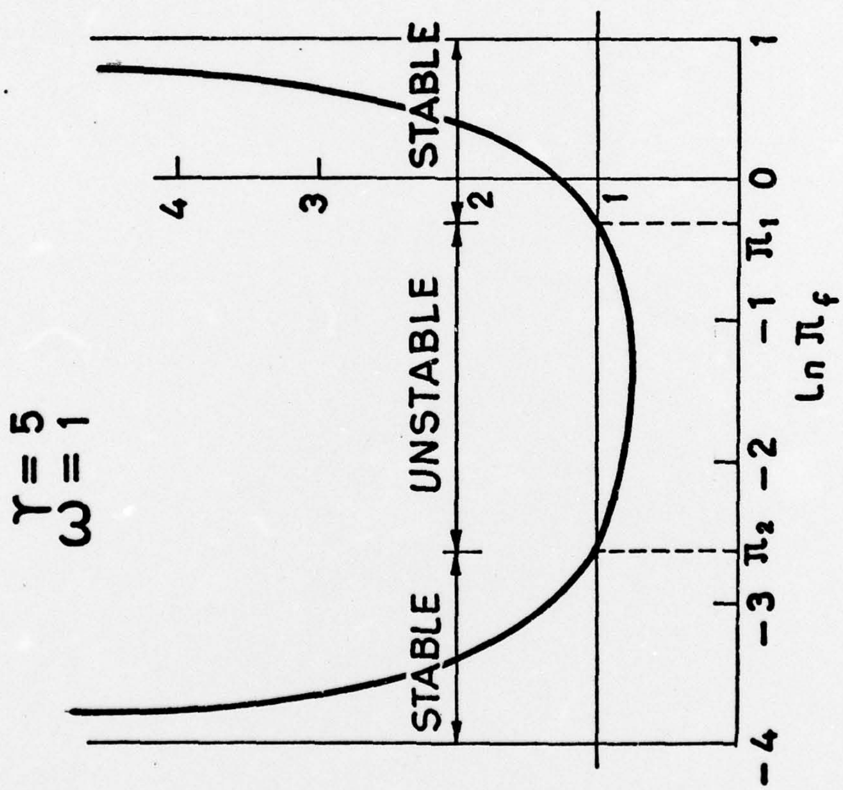

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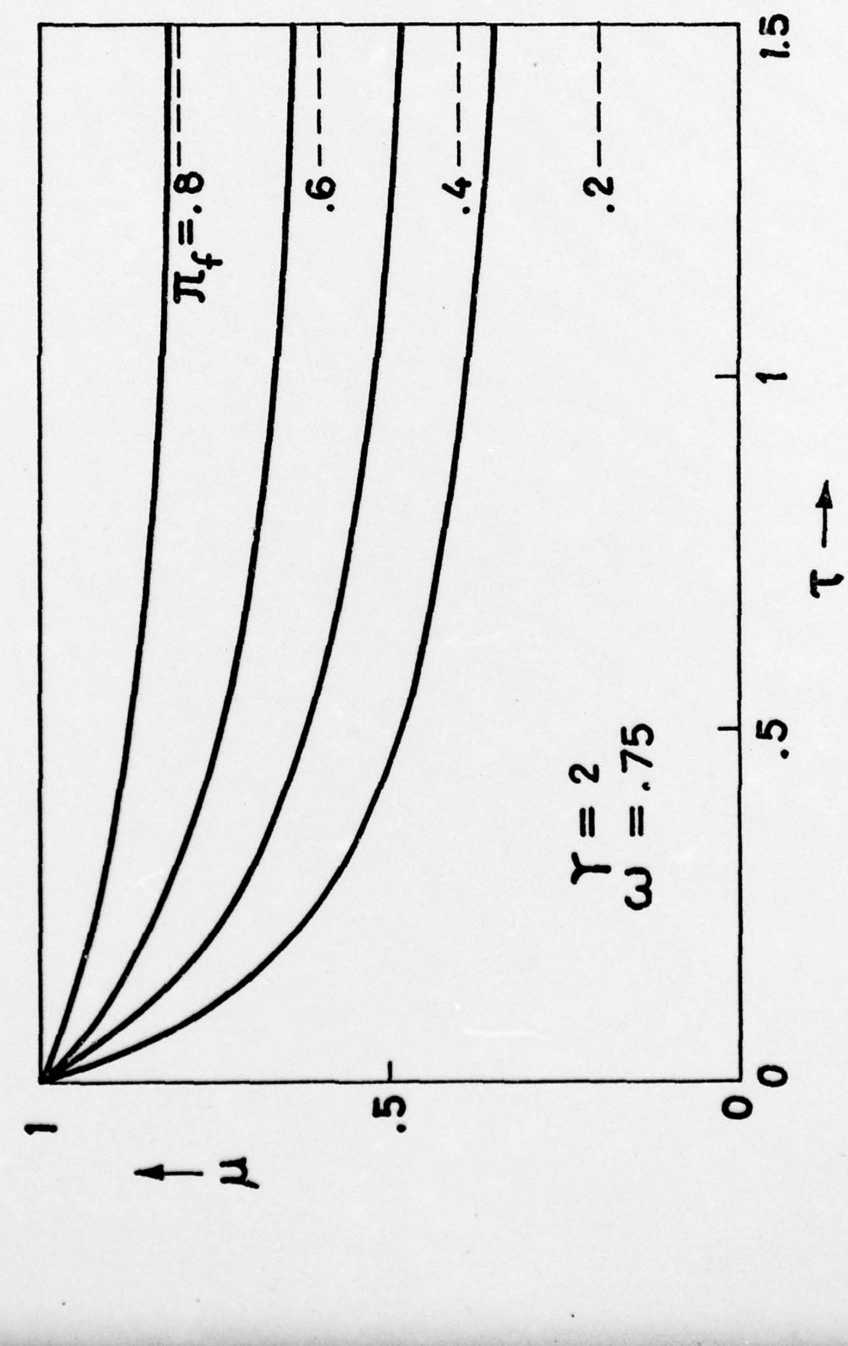

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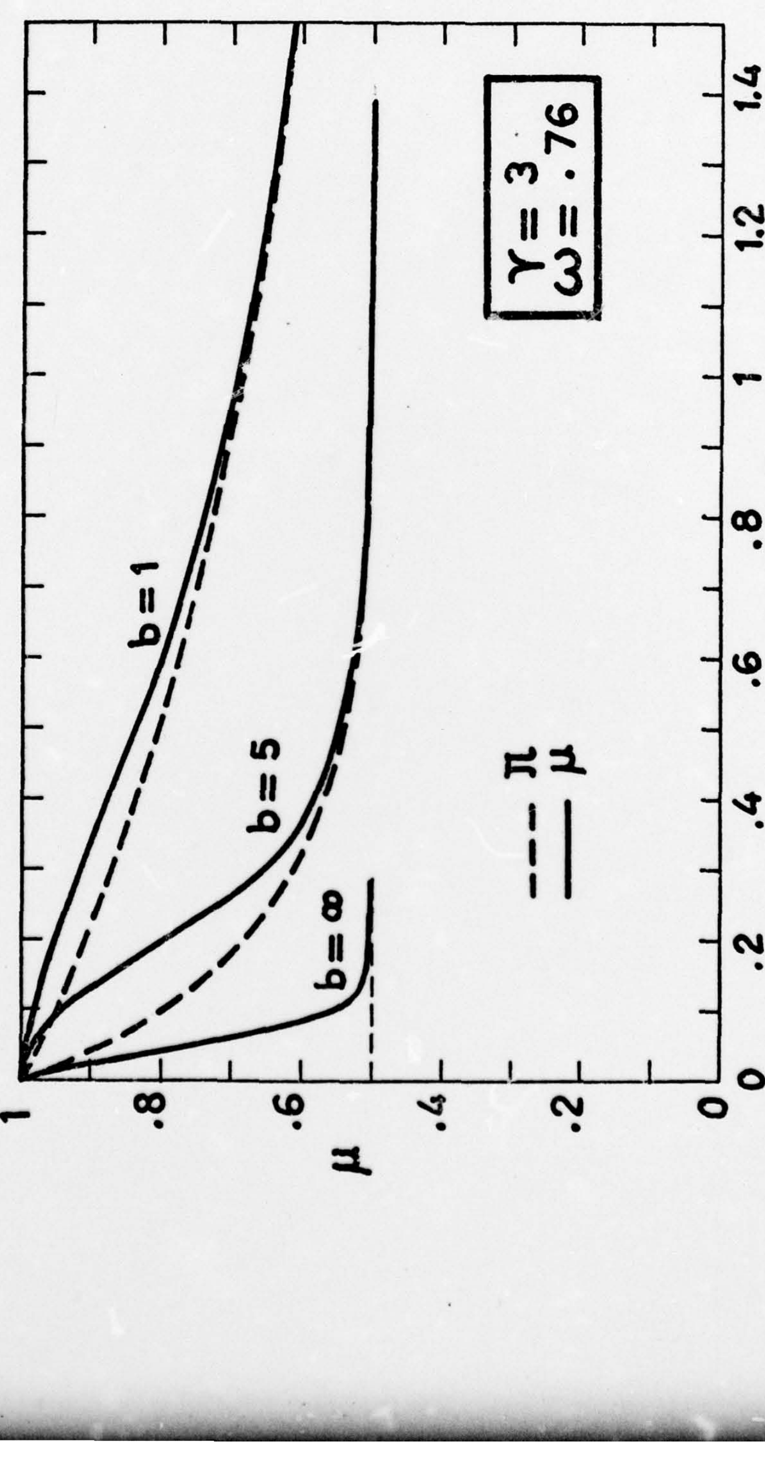

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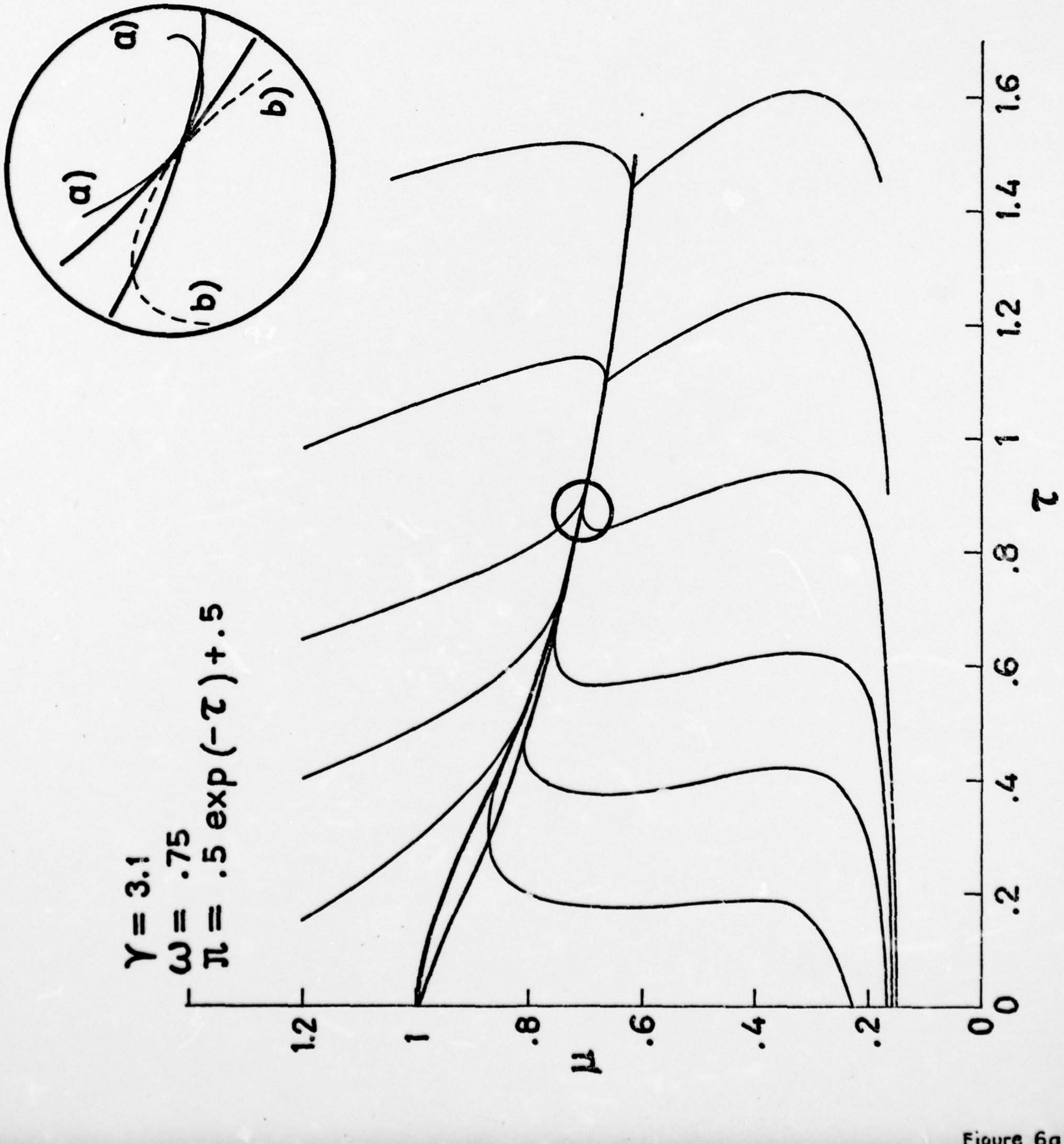

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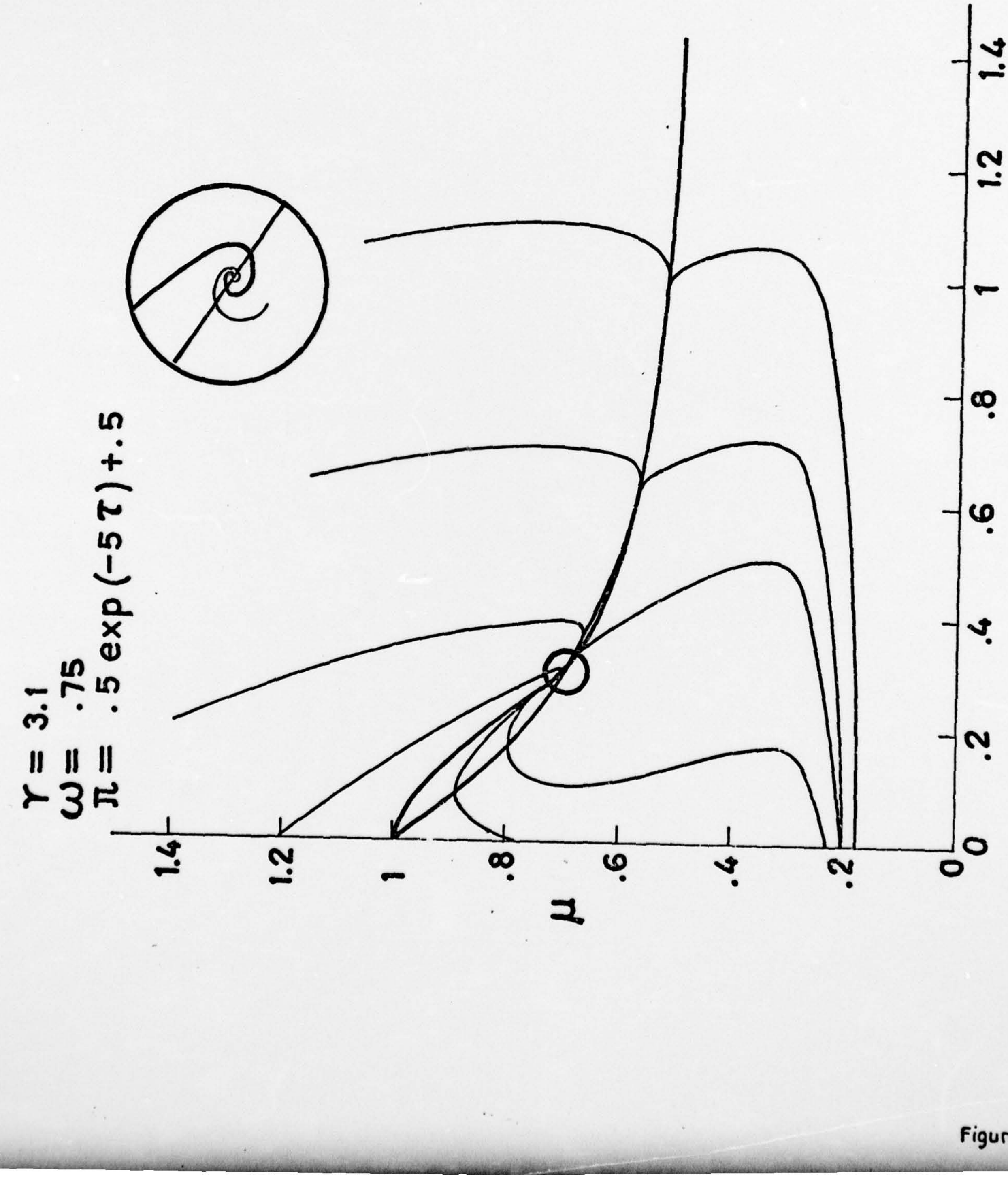
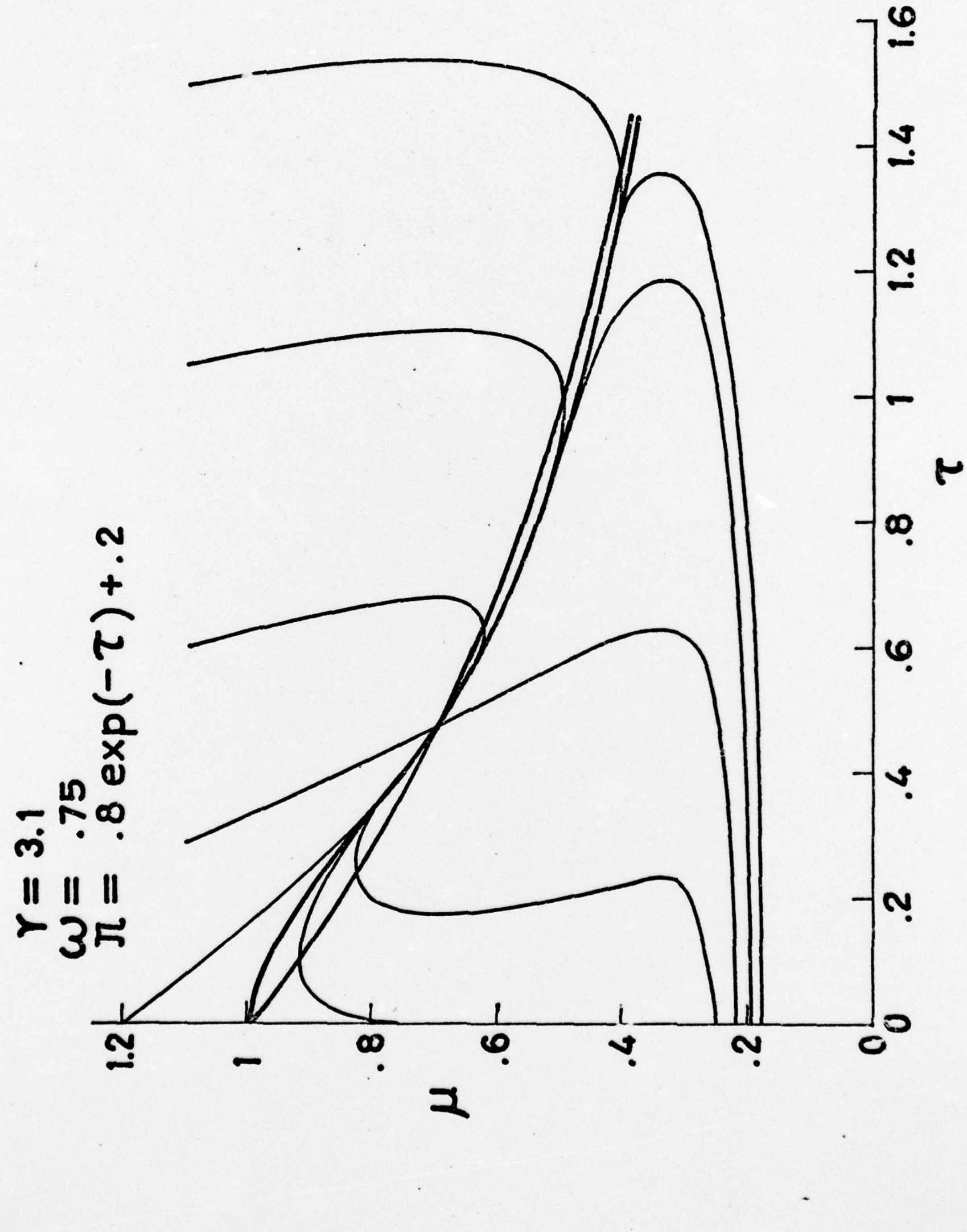


Figure 6b



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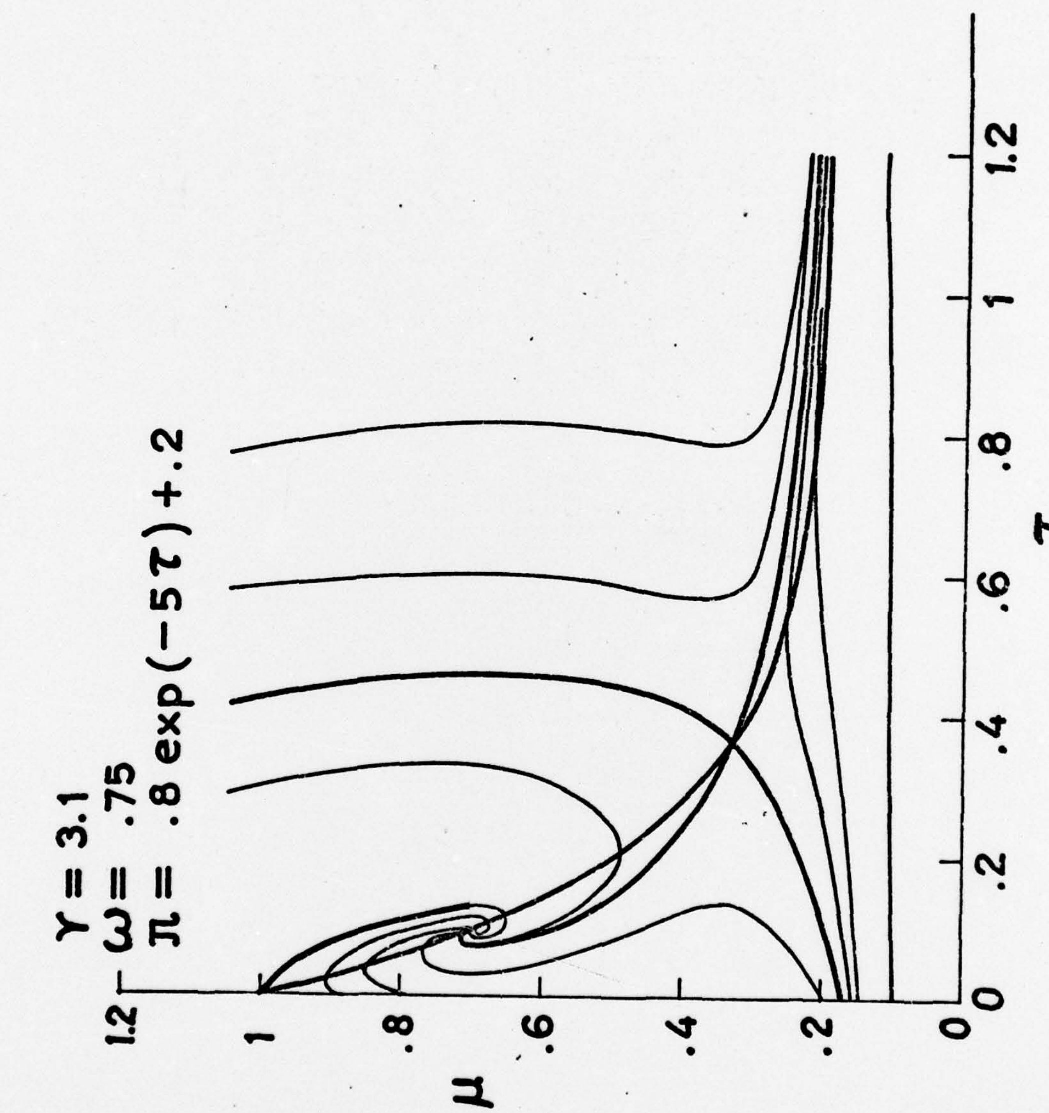
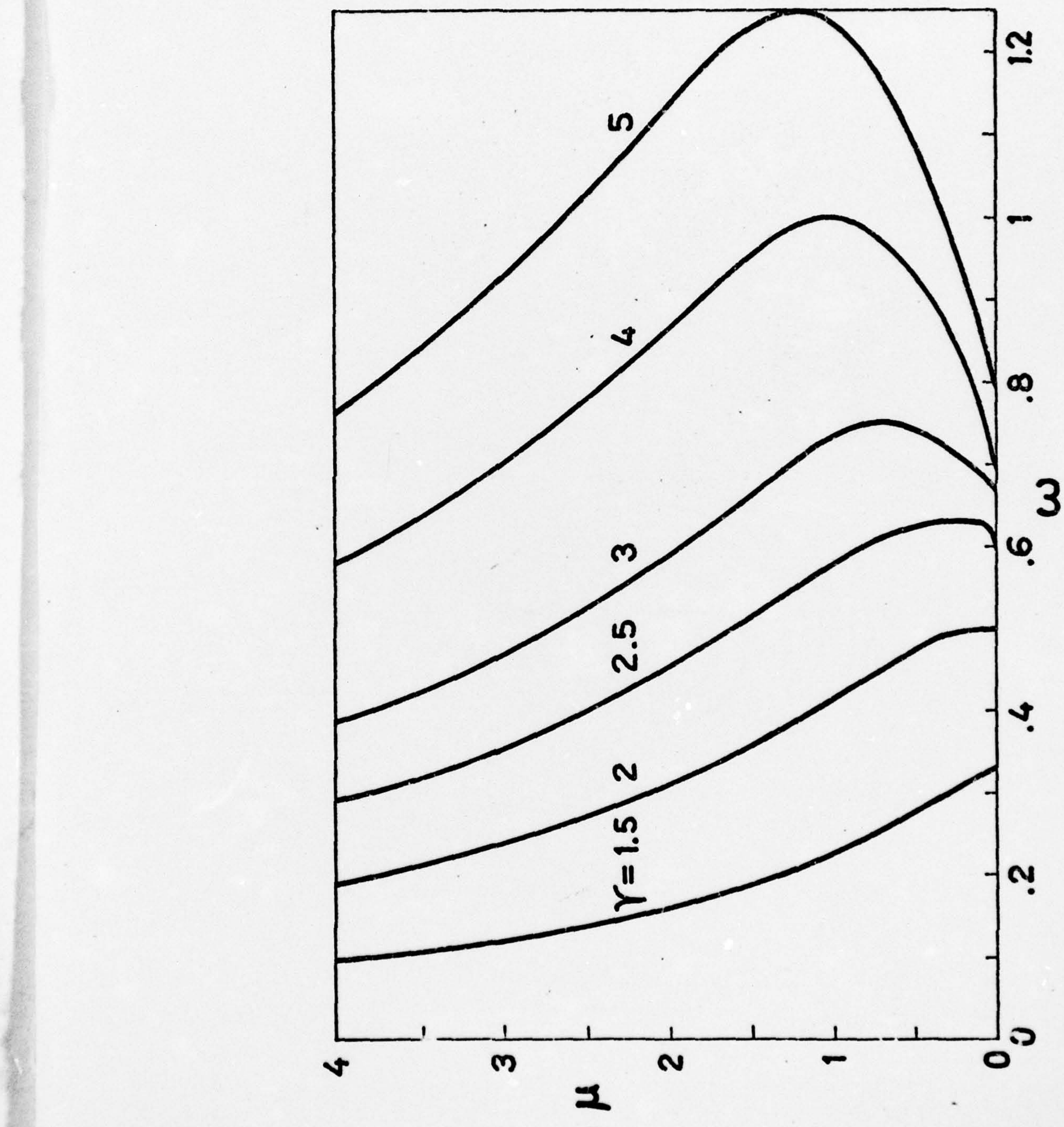


Figure 7b



Figure